LIBERTY PAPER SET

STD. 12 : Mathematics

Full Solution

Time : 3 Hours **ASSIGNMENT PAPER 4**

PART A

1. (B) **2.** (C) **3.** (D) **4.** (C) **5.** (D) **6.** (C) **7.** (C) **8.** (A) **9.** (D) **10.** (A) **11.** (D) **12.** (C) **13.** (B) **14.** (B) **15.** (A) **16.** (C) **17.** (B) **18.** (C) **19.** (D) **20.** (B) **21.** (B) **22.** (B) **23.** (B) **24.** (D) **25.** (B) **26.** (C) **27.** (B) **28.** (C) **29.** (C) **30.** (C) **31.** (C) **32.** (A) **33.** (D) **34.** (A) **35.** (C) **36.** (B) **37.** (D) **38.** (B) **39.** (B) **40.** (B) **41.** (A) **42.** (C) **43.** (C) **44.** (B) **45.** (A) **46.** (D) **47.** (B) **48.** (A) **49.** (D) **50.** (B)

PART B

SECTION A

1.

Here, Take $sin^{-1}\frac{3}{5} = \alpha$ and $tan^{-1}\frac{17}{31} = \beta$

$\therefore \quad sin\ \alpha = \frac{3}{5}$ and $tan\ \beta = \frac{17}{31}$

$$tan\ \alpha = \frac{3}{5}$$

$$tan\ 2\alpha = \frac{2\ tan\ \alpha}{1 - tan^2\alpha}$$

$$= \frac{2\left(\frac{3}{4}\right)}{1 - \frac{9}{16}}$$

$$= \frac{\frac{3}{2}}{\frac{7}{16}}$$

$$= \frac{24}{7}$$

$$tan\ (2\alpha - \beta) = \frac{tan\ 2\alpha - tan\ \beta}{1 + tan\ 2\alpha\ tan\ \beta}$$

$$= \frac{\frac{24}{7} - \frac{17}{31}}{1 + \frac{24}{7} \times \frac{17}{31}}$$

$$= \frac{\frac{744 - 119}{217}}{\frac{217 + 408}{217}}$$

$$= \frac{605}{605}$$

$$tan\ (2\alpha - \beta) = 1$$

$$\therefore \quad tan\ (2\alpha - \beta) = tan\ \frac{\pi}{4}$$

$$\therefore \quad 2\alpha - \beta = \frac{\pi}{4}$$

$$\therefore \quad 2\ sin^{-1}\frac{3}{5} - tan^{-1}\frac{17}{31} = \frac{\pi}{4}$$

2.

$$tan^{-1}\left(\frac{\sqrt{1+x^2} - 1}{x}\right)$$

Suppose, $x = tan\ \theta$

$\theta = tan^{-1}\ x,\ \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$= tan^{-1}\left(\frac{\sqrt{1 + tan^2\theta} - 1}{tan\ \theta}\right)$$

$$= tan^{-1}\left(\frac{\sqrt{sec^2\theta - 1}}{tan\ \theta}\right)$$

$$= tan^{-1}\left(\frac{|sec\ \theta| - 1}{tan\ \theta}\right)$$

$$= tan^{-1}\left(\frac{sec\ \theta - 1}{tan\ \theta}\right) \quad \left(\because -\frac{\pi}{2} < \theta < \frac{\pi}{2} \Rightarrow sec\theta > 0\right)$$

$$= tan^{-1}\left(\frac{\frac{1}{cos\ \theta} - 1}{\frac{sin\ \theta}{cos\ \theta}}\right)$$

$$= tan^{-1}\left(\frac{1 - cos\ \theta}{sin\ \theta}\right)$$

$$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$= \tan^{-1}\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)$$

$$= \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{\theta}{2} \qquad \left(\because \; -\frac{\pi}{2} < \theta < \frac{\pi}{2} \Rightarrow -\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{4}\right)$$

$$= \frac{1}{2} \cdot \tan^{-1} x$$

3.

Now, differentiate w.r.t. x,

$$2\sin x \cdot \cos x + 2\cos y\,(-\sin y)\frac{dy}{dx} = 0$$

$$\therefore \; \sin 2x - \sin 2y\frac{dy}{dx} = 0$$

$$\therefore \; \frac{dy}{dx}(\sin 2y) = \sin 2x$$

$$\therefore \; \frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}$$

4.

$$I = \int \sqrt{1-4x^2}\; dx$$

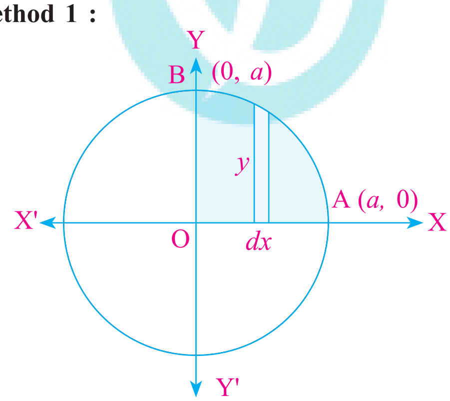
$$= \int \sqrt{(1)^2 - (2x)^2}$$

$$= \frac{2x}{2\times 2}\sqrt{1-4x^2} + \frac{1}{2}\,\frac{\sin^{-1}(2x)}{2} + c$$

$$I = \frac{x}{2}\sqrt{1-4x^2} + \frac{\sin^{-1}(2x)}{4} + c$$

5.

Method 1 :



From Fig, the whole area enclosed by the given circle = 4 (area of the region AOBA bounded by the curve, x-axis and the ordinates $x = 0$ and $x = a$) [as the circle is symmetrical about both x-axis and y-axis]

$$= 4\int_0^a y\,dx \quad \text{(taking vertical strips)}$$

$$= 4\int_0^a \sqrt{a^2-x^2}\;dx$$

Since, $x^2 + y^2 = a^2$ gives $y = \pm\sqrt{a^2-x^2}$

As the region AOBA lies in the first quadrant, y is taken as positive. Integrating, we get the whole area enclosed by the given circle

$$= 4\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

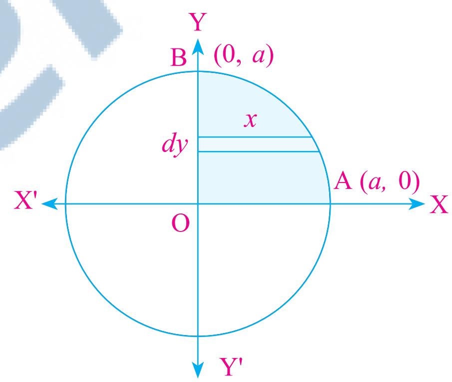
$$= 4\left[\left(\frac{a}{2}\times 0 + \frac{a^2}{2}\sin^{-1}1\right) - 0\right]$$

$$= 4\left(\frac{a^2}{2}\right)\left(\frac{\pi}{2}\right)$$

$= \pi a^2$ sq. unit

Method 2 :

considering horizontal strips as shown in Fig, the whole area of the region enclosed by circle



$$= 4\int_0^a x\,dy$$

$$= 4\int_0^a \sqrt{a^2-y^2}\;dy$$

$$= 4\left[\frac{y}{2}\sqrt{a^2-y^2} + \frac{a^2}{2}\sin^{-1}\frac{y}{a}\right]_0^a$$

$$= 4\left[\left(\frac{a}{2}\times 0 + \frac{a^2}{2}\sin^{-1}1\right) - 0\right]$$

$$= 4\,\frac{a^2}{2}\,\frac{\pi}{2}$$

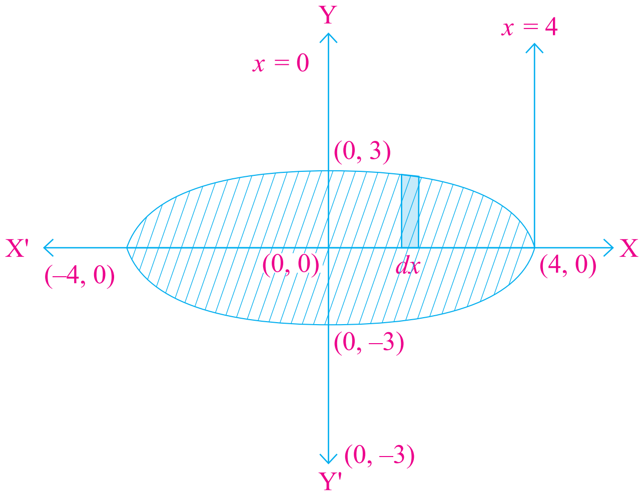
$= \pi a^2$ sq. unit

6.

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$a^2 = 16$, $a = 4$ $(a > b)$

$b^2 = 9$, $b = 3$



Required Area :

A = 4 × Area bounded in the first quadrant

$\therefore$ A = 4|I|

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$\therefore\ y^2 = 9\left(1 - \frac{x^2}{16}\right)$$

$$\therefore\ y^2 = \frac{9}{16}\ (16 - x^2)$$

$$\therefore\ y = \frac{3}{4}\sqrt{16 - x^2}$$

$$I = \int_0^4 y\ dx$$

$$I = \int_0^4 \frac{3}{4}\sqrt{16 - x^2}\ dx$$

$$I = \frac{3}{4}\int_0^4 \sqrt{16 - x^2}\ dx$$

$$= \frac{3}{4}\int_0^4 \sqrt{4^2 - x^2}\ dx$$

$$I = \frac{3}{4}\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$$

$$I = \frac{3}{4}\left[\left(\frac{4}{2}(0) + 8\sin^{-1}(1)\right) - \left(0 + \sin^{-1}(0)\right)\right]$$

$$I = \frac{3}{4}\left(8 \cdot \frac{\pi}{2}\right)$$

I = 3π

Now, A = 4|I|

= $4|3\pi|$

$\therefore$ A = 12π sq. units

7.

$(1 + e^{2x})\ dy + (1 + y^2)\ e^x\ dx = 0$

$\therefore\ (1 + e^{2x})\ dy = -(1 + y^2)\ e^x\ dx$

$$\therefore\ \frac{dy}{dx} = \frac{-(1 + y^2)\,e^x}{(1 + e^{2x})}$$

$$\therefore\ \frac{dy}{(1 + y^2)} = \frac{-dx\ e^x}{1 + e^{2x}}$$

$\to$ Integrate both the sides,

$$\therefore\ \int \frac{dy}{y^2 + 1} = -\int \frac{e^x\ dx}{(e^x)^2 + 1}$$

Take, $e^x = t$

$\therefore\ e^x\ dx = dt$

$$\therefore\ \int \frac{dy}{y^2 + 1} = -\int \frac{dt}{t^2 + 1}$$

$\therefore\ \tan^{-1}(y) = -\tan^{-1}(t) + c$

$\therefore\ \tan^{-1}(y) = -\tan^{-1}(e^x) + c$

$\therefore\ \tan^{-1}(y) + \tan^{-1}(e^x) = c$ ... (1)

$\to$ If $x = 0$ and $y = 1$ then,

$\therefore\ \tan^{-1}(1) + \tan^{-1}(e^0) = c$

$\therefore\ \tan^{-1}(1) + \tan^{-1}(1) = c$

$\therefore\ 2\ \tan^{-1}(1) = c$

$\therefore\ 2 \cdot \frac{\pi}{4} = c$

$\therefore\ c = \frac{\pi}{2}$

$\to$ Put the value of c in equation (1),

$\therefore\ \tan^{-1}(y) + \tan^{-1}(e^x) = \frac{\pi}{2}$;

Which is required particular solution of given differnetial equation.

8.

$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{b} = \hat{i} - 2\hat{j} - \hat{k}$

Resultant vector of $\vec{a}$ and $\vec{b}$,

$\vec{a} + \vec{b} = 3\hat{i} + \hat{j}$

$|\vec{a} + \vec{b}| = \sqrt{9 + 1}$

$= \sqrt{10}$

Parellel vector to the resultant vectors $\vec{a}$ and $\vec{b}$ with magnitude of 5.

$$= \frac{5(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|}$$

$$= \frac{5(3\hat{i} + \hat{j})}{\sqrt{10}}$$

$$= \frac{15}{\sqrt{10}}\hat{i} + \frac{5}{\sqrt{10}}\hat{j}$$

$$= \frac{(5 \times 3)\sqrt{10}}{10}\ \hat{i} + \frac{5\sqrt{10}}{10}\ \hat{j}$$

$$= \frac{3}{2}\sqrt{10}\ \hat{i} + \frac{1}{2}\sqrt{10}\ \hat{j}$$

9.

Direction cosine of $L_1 = \frac{12}{13}\hat{i} - \frac{3}{13}\hat{j} - \frac{4}{13}\hat{k}$

Direction cosine $L_2 = \frac{4}{13}\hat{i} + \frac{12}{13}\hat{j} + \frac{3}{13}\hat{k}$

Direction cosine $L_3 = \frac{3}{13}\hat{i} - \frac{4}{13}\hat{j} + \frac{12}{13}\hat{k}$

L_1 is parallel to vector $\overrightarrow{b_1} = 12\hat{i} - 3\hat{j} - 4\hat{k}$

L_2 is parallel to vector $\overrightarrow{b_2} = 4\hat{i} + 12\hat{j} + 3\hat{k}$

L_3 is parallel to vector $\overrightarrow{b_3} = 3\hat{i} - 4\hat{j} + 12\hat{k}$

Now, $\overrightarrow{b_1} \cdot \overrightarrow{b_2}$

$= (12\hat{i} - 3\hat{j} - 4\hat{k})\cdot(4\hat{i} + 12\hat{j} + 3\hat{k})$

$= 48 - 36 - 12$

$= 0$

$\overrightarrow{b_2} \cdot \overrightarrow{b_3}$

$= (4\hat{i} + 12\hat{j} + 3\hat{k})\cdot(3\hat{i} - 4\hat{j} + 12\hat{k})$

$= 12 - 48 + 36$

$= 0$

$\overrightarrow{b_3} \cdot \overrightarrow{b_1}$

$= (3\hat{i} - 4\hat{j} + 12\hat{k})\cdot(12\hat{i} - 3\hat{j} - 4\hat{k})$

$= 36 + 12 - 48$

$= 0$

$\overrightarrow{b_1}, \overrightarrow{b_2}, \overrightarrow{b_3}$ perpendicular to each other.

$\therefore$ L_1, L_2, L_3 are perpendicular lines.

10.

The direction ratios of the first line are 3, 5, 4 and the direction ratios of the second line are 1, 1, 2. If θ is the angle between them, then

$$\cos\theta = \left|\frac{3\cdot 1 + 5\cdot 1 + 4\cdot 2}{\sqrt{3^2+5^2+4^2}\,\sqrt{1^2+1^2+2^2}}\right|$$

$$= \frac{16}{\sqrt{50}\,\sqrt{6}}$$

$$= \frac{16}{5\sqrt{2}\,\sqrt{6}}$$

$$= \frac{8\sqrt{3}}{15}$$

Hence, the required angle is $\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$.

11.

A fair coin and an unbiased die are forced,

S = {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6),
(T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}

$n = 12$

Event A : Head appears on the coin

A = {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)}

$\therefore r = 6$

$$\therefore P(A) = \frac{r}{n} = \frac{6}{12} = \frac{1}{2}$$

Event B : 3 on the die.

B = {(H, 3), (T, 3)}

$\therefore r = 2$

$$\therefore P(B) = \frac{r}{n} = \frac{2}{12} = \frac{1}{6}$$

$\therefore$ A ∩ B = {(H, 3)}

$\therefore r = 1$

$$\therefore P(A \cap B) = \frac{1}{12} \quad \text{....... (i)}$$

$$\therefore P(A)\cdot P(B) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12} \quad \text{....... (ii)}$$

$\therefore$ $P(A \cap B) = P(A) \cdot P(B)$

A and B are independent events.

12.

$P(A) = \frac{1}{4}$ $\quad P(B) = \frac{1}{2}$

$P(A \cap B) = \frac{1}{8}$

P(Not A and Not B)

$= P(A' \cap B')$

$= P(A \cup B)'$

$= 1 - P(A \cup B)$

$= 1 - P(A) - P(B) + P(A \cap B)$

$= 1 - \left[\frac{1}{4} - \frac{1}{2} + \frac{1}{8}\right]$

$= \frac{3}{8}$

SECTION B

13.

Here, S = $\{(a, b) : a \le b^2\}$

Suppose, $(a, a) \in S$, $\forall a \in R$

$\therefore$ $a \le a^2$ which is not possible

$\therefore$ $(a, a) \notin S$

$\therefore$ S is not reflexive.

Example for : $\left(\frac{1}{3}, \frac{1}{3}\right)$, $\frac{1}{3} \le \frac{1}{9}$ which is not possible

$\therefore$ $\left(\frac{1}{3}, \frac{1}{3}\right) \notin S$

Suppose, $(2, 5) \in S$ But $(5, 2) \notin S$

$\therefore$ For (5, 2), $5 \le 4$ which is not possible

$\therefore$ S is not symmetric.

Suppose, $(a, b) \in S$ and $(b, c) \in S$

$\therefore$ $a \le b^2$ and $b \le c^2$

$\therefore$ $b^2 \le c^4$

$\therefore$ $a \le b^2 \le c^4$

$\therefore$ $a \le c^4$

$\therefore$ $(a, c) \notin S$

$\therefore$ S is not transitive.

Thus, Relation S is not reflexive, symmetric, transitive.

14.

Total number of Chemistry books = 10 dozen

= 10×12

= 120

Total number of Physics books = 8 dozen

= 8×12

= 96

Total number of Economics books = 10 dozen

= 10×12

= 120

Selling price of Chemistry book is ₹ 80

Selling price of Physics book is ₹ 60

Selling price of Economics book is ₹ 40

Total cost price,

$$= [120 \quad 96 \quad 120]\begin{bmatrix}80\\60\\40\end{bmatrix}$$

$$= [9600 + 5760 + 4800]$$

$$= [20160]$$

The total amount of the bookshop will receive from selling all the books is ₹ 20,160.

15.

$$A^2 = A \cdot A = \begin{bmatrix}3 & 2\\1 & 1\end{bmatrix}\begin{bmatrix}3 & 2\\1 & 1\end{bmatrix} = \begin{bmatrix}9+2 & 6+2\\3+1 & 2+1\end{bmatrix}$$

$$\therefore \quad A^2 = \begin{bmatrix}11 & 8\\4 & 3\end{bmatrix}$$

$$A^2 + aA + bI = O$$

$$\therefore \begin{bmatrix}11 & 8\\4 & 3\end{bmatrix} + a\begin{bmatrix}3 & 2\\1 & 1\end{bmatrix} + b\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$$\therefore \begin{bmatrix}11 & 8\\4 & 3\end{bmatrix} + \begin{bmatrix}3a & 2a\\a & a\end{bmatrix} + \begin{bmatrix}b & 0\\0 & b\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$$\therefore \begin{bmatrix}11+3a+b & 8+2a\\4+a & 3+a+b\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$11 + 3a + b = 0$ .... (1)

$8 + 2a = 0$

$a + 4 = 0$

$\therefore$ $a = -4$

Put $a = -4$ in equation (1),

$\therefore$ $11 + 3(-4) + b = 0$

$\therefore$ $11 - 12 + b = 0$

$\therefore$ $b = 1$

Thus, $a = -4$, $b = 1$

16.

$$y = x^{x^2-3} + (x-3)^{x^2}$$

Suppose, $u = x^{(x^2-3)}$

$v = (x-3)^{x^2}$

$\therefore y = u + v$

Differentiate w.r.t. x,

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{...... (1)}$$

Here, $u = x^{(x^2-3)}$

Take both the sides *log*,

$\log u = (x^2 - 3) \log x$

Differentiate w.r.t. x,

$$\frac{1}{u}\frac{du}{dx} = (x^2-3)\frac{1}{x} + \log x \,.\, (2x)$$

$$\therefore \frac{du}{dx} = u\left[\frac{x^2-3}{x} + 2x \cdot \log x\right]$$

$$\therefore \frac{du}{dx} = x^{(x^2-3)}\left(\frac{x^2-3}{x} + 2x \cdot \log x\right) \quad \text{...... (2)}$$

But, $v = (x-3)^{x^2}$

Take both the sides *log*,

$\log v = x^2 \log(x-3)$

Differentiate w.r.t. x,

$$\frac{1}{v}\frac{dv}{dx} = \frac{x^2}{(x-3)}(1) + \log(x-3) \cdot (2x)$$

$$\therefore \frac{dv}{dx} = v\left[\frac{x^2}{x-3} + 2x \cdot \log(x-3)\right]$$

$$\therefore \frac{dv}{dx} = (x-3)^{x^2}\left(\frac{x^2}{x-3} + 2x \cdot \log(x-3)\right) \quad \text{...... (3)}$$

Put the value of equation (2) and (3) in equation (1),

$$\frac{dy}{dx} = x^{(x^2-3)}\left(\frac{x^2-3}{x} + 2x \log x\right) + (x-3)^{x^2}\left(\frac{x^2}{x-3} + 2x \log(x-3)\right), \; x > 3$$

17.

We have

$f(x) = 2x^3 - 6x^2 + 6x + 5$

or $f'(x) = 6x^2 - 12x + 6$

$= 6(x-1)^2$

or $f'(x) = 0$ at $x = 1$

Thus, $x = 1$ is the only critical point of f. We shall now examine this point for local maxima and/or local minima of f. Observe that $f'(x) \geq 0$, for all $x \in R$ and in particular $f'(x) > 0$, for values close to 1 and to the left and to the right of 1.

Therefore, by first derivative test, the point $x = 1$ is neither a point of local maxima nor a point of local minima.

Liberty

Hence $x = 1$ is a point of inflexion.

Remark : One may note that since $f'(x)$, in Example 30, never changes its sign on R, graph of f has no turning points and hence no point of local maxima or local minima.

We shall now give another test to examine local maxima and local minima of a given function. This test is often easier to apply than the first derivative test.

18.

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

$2\vec{a} - \vec{b} + 3\vec{c}$

$= 2\hat{i} + 2\hat{j} + 2\hat{k} - 2\hat{i} + \hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}$

$= 3\hat{i} - 3\hat{j} + 2\hat{k}$

Unit parallel vector to the vector $2\vec{a} - \vec{b} + 3\vec{c}$,

$$= \frac{2\vec{a} - \vec{b} + 2\vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}\,|}$$

$$= \frac{3\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{9+9+4}}$$

$$= \frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}\hat{k}$$

19.

The two lines are parallel

We have $\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$,

$\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$ and

$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

Therefore, the distance between the lines is given by,

$$d = \left| \frac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}\,|} \right|$$

$$= \left| \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}}{\sqrt{4+9+36}} \right|$$

$$= \frac{|-9\hat{i} + 14\hat{j} - 4\hat{k}|}{\sqrt{49}}$$

$$= \frac{\sqrt{293}}{\sqrt{49}}$$

$$= \frac{\sqrt{293}}{7} \text{ unit}$$

20.

$x + 2y \le 10$

$3x + y \le 15$

Objective function $Z = 3x + 2y$

$x \ge 0$

$y \ge 0$

$x + 2y = 10$ ... (i)

x	0	10
y	5	0

$3x + y = 15$ ... (ii)

x	0	5
y	15	0

Solving equation (i) and (ii),

$$\begin{array}{r} x + 2y = 10 \\ 6x + 2y = 30 \\ - \quad - \quad - \\ \hline -5x = -20 \end{array}$$

$\therefore x = 4$ $\qquad$ $\therefore y = 3$

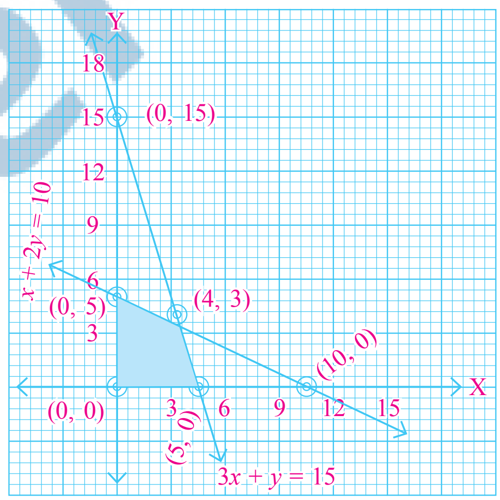
(4, 3) $\qquad$ (0, 0)

Put $x = 4$ in eq^n (i),

$4 + 2y = 10$

$\therefore 2y = 6$

$\therefore y = 3$



The shaded region in fig is feasible region determined by the system of constraints which is bounded. The co-ordinates of corner points are (0, 0), (6, 0), (4, 3) and (0, 5).

Corner Point	Corresponding value of $Z = 3x + 2y$
(0, 5)	10
(5, 0)	15
(4, 3)	18 ← Maximum
(0, 0)	0

Thus, the Maximum value of Z is 18 at point (4, 3).

21.

Let E be the event that the doctor visits the patient late and let T_1, T_2, T_3, T_4 be the events that the doctor comes by train, bus, scooter, and other means of transport respectively.

Then, $P(T_1) = \frac{3}{10}$,

$P(T_2) = \frac{1}{5}$,

$P(T_3) = \frac{1}{10}$

and $P(T_4) = \frac{2}{5}$ (given)

$P(E \mid T_1)$ = Probability that the doctor arriving late comes by train $= \frac{1}{4}$

Similarly, $P(E \mid T_2) = \frac{1}{3}$,

$P(E \mid T_3) = \frac{1}{12}$,

$P(E \mid T_4) = 0$

since he is not late if he comes by other means of transport.

Therefore, by Bayes' Theorem, we have,

$P(T_1 \mid E)$ = Probability that the doctor arriving late comes by train

$$= \frac{P(T_1)\cdot P(E|T_1)}{P(T_1)P(E|T_1)+P(T_2)P(E|T_2)+P(T_3)P(E|T_3)+P(T_4)P(E|T_4)}$$

$$= \frac{\frac{3}{10}\times\frac{1}{4}}{\frac{3}{10}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{3}+\frac{1}{10}\times\frac{1}{12}+\frac{2}{5}\times 0}$$

$$= \frac{3}{40}\times\frac{120}{18} = \frac{1}{2}$$

Hence, the required probability is $\frac{1}{2}$.

SECTION C

22.

(a) If unit sale prices of x, y and z are Rs. 2.50, Rs. 1.50 and Rs. 1.00 respectively.

∴ Total Revenue in Market-I can be written as matrix form

$$[10000 \quad 2000 \quad 18000]\begin{bmatrix}2.50\\1.50\\1.00\end{bmatrix}$$

$= [10000 \times 2.50 + 2000 \times 1.50 + 18000 \times 1]$

$= [25000 + 3000 + 18000]$

$= [46000]$

∴ Total Revenue in Market-I is Rs. 46,000.

Total Revenue in Market-II can be written as matrix form.

$$[6000 \quad 20000 \quad 8000]\begin{bmatrix}2.50\\1.50\\1.00\end{bmatrix}$$

$= [6000 \times 2.50 + 20000 \times 1.50 + 8000 \times 1.00]$

$= [15000 + 30000 + 8000]$

$= [53000]$

∴ Total Revenue in Market-II is Rs. 53,000.

(b) If the unit sale prices of x, y and z are Rs. 2, Rs. 1 and Rs. 0.50 respectively.

Total cost of Market-I

$$[10000 \quad 2000 \quad 18000]\begin{bmatrix}2\\1\\0.50\end{bmatrix}$$

$= [10000 \times 2 + 2000 \times 1 + 18000 \times 0.50]$

$= [20000 + 2000 + 9000]$

$= [31000]$

∴ Total cost of Market-I is Rs. 31,000.

∴ Total cost of Market-II :

$$= [6000 \quad 20000 \quad 8000]\begin{bmatrix}2\\1\\0.50\end{bmatrix}$$

$= [12000 + 20000 + 4000]$

$= [36000]$

∴ Profit of Market-I

= S.P. – C.P.

= 46000 – 31000

= Rs. 15000

∴ Profit of Market-II

= S.P. – C.P.

= 53000 – 36000

= Rs. 17,000

23.

The equation can be represented as matrix form,

$$\therefore \begin{bmatrix}2 & 3 & 3\\1 & -2 & 1\\3 & -1 & -2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}5\\-4\\3\end{bmatrix}$$

∴ AX = B

Where, $A = \begin{bmatrix}2 & 3 & 3\\1 & -2 & 1\\3 & -1 & -2\end{bmatrix}$, $X = \begin{bmatrix}x\\y\\z\end{bmatrix}$, $B = \begin{bmatrix}5\\-4\\3\end{bmatrix}$

AX = B

∴ $X = A^{-1}B$

For finding A^{-1},

$$|A| = \begin{vmatrix}2 & 3 & 3\\1 & -2 & 1\\3 & -1 & -2\end{vmatrix}$$

$= 2(4 + 1) - 3(-2 - 3) + 3(-1 + 6)$

$= 2(5) - 3(-5) + 3(5)$

$= 10 + 15 + 15$

$= 40 \neq 0$

We get unique solution

For finding *adj* A,

Co-factor of element 2 $A_{11} = (-1)^2 \begin{vmatrix} -2 & 1 \\ -1 & -2 \end{vmatrix}$

$= 1(4 + 1)$

$= 5$

Co-factor of element 3 $A_{12} = (-1)^3 \begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix}$

$= (-1)(-2 - 3)$

$= 5$

Co-factor of element 3 $A_{13} = (-1)^4 \begin{vmatrix} 1 & -2 \\ 3 & -1 \end{vmatrix}$

$= 1(-1 + 6)$

$= 5$

Co-factor of element 1 $A_{21} = (-1)^3 \begin{vmatrix} 3 & 3 \\ -1 & -2 \end{vmatrix}$

$= (-1)(-6 + 3)$

$= 3$

Co-factor of element –2 $A_{22} = (-1)^4 \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix}$

$= 1(-4 - 9)$

$= -13$

Co-factor of element 1 $A_{23} = (-1)^5 \begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix}$

$= (-1)(-2 - 9)$

$= 11$

Co-factor of element 3 $A_{31} = (-1)^4 \begin{vmatrix} 3 & 3 \\ -2 & 1 \end{vmatrix}$

$= 1(3 + 6)$

$= 9$

Co-factor of element –1 $A_{32} = (-1)^5 \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}$

$= (-1)(2 - 3)$

$= 1$

Co-factor of element –2 $A_{33} = (-1)^6 \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix}$

$= 1(-4 - 3)$

$= -7$

$$adj\,A = \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\, adj\,A$$

$$= \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$X = A^{-1}B$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 25 - 12 + 27 \\ 25 + 52 + 3 \\ 25 - 44 - 21 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Solution : $x = 1, y = 2, z = -1$

24.

$y = cos^{-1}x$

$\therefore\ x = cosy$

Now, differentiate again w.r.t. y,

$$\frac{dx}{dy} = -\,siny$$

$$\therefore \frac{dy}{dx} = \frac{-1}{siny}$$

$$\therefore \frac{dy}{dx} = -\,cosecy \qquad \text{........ (1)}$$

Now, differentiate again w.r.t. x,

$$\therefore \frac{d^2y}{dx^2} = -\,[-\,cosecy \cdot coty]\,\frac{dy}{dx}$$

$$= cosecy \cdot coty\ [-\,cosecy] \quad (\because \text{From (1)})$$

$$\therefore \frac{d^2y}{dx^2} = -\,cosec^2\,y \cdot cot\,y$$

25.

Let r, h and a be as in Fig. Then $tan\ \alpha = \frac{r}{h}$.

So $\alpha = tan^{-1}\left(\frac{r}{h}\right)$

But $\alpha = tan^{-1}(0.5)$ (given)

or $\frac{r}{h} = 0.5$

or $r = \frac{h}{2}$

Let V be the volume of the cone. Then

$$\therefore\ V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$$

$$= \frac{\pi h^3}{12}$$

Therefore, $\frac{dV}{dt} = \frac{d}{dh}\left(\frac{\pi h^3}{12}\right)\cdot\frac{dh}{dt}$ (by Chain Rule)

$= \frac{\pi}{4}h^2\cdot\frac{dh}{dt}$

Now rate of change of volume,

i.e., $= \frac{dV}{dt} = 5$ m^3/h and $h = 4$ m.

THerefore, $5 = \frac{\pi}{4}(4)^2\cdot\frac{dh}{dt}$

Or $\frac{dh}{dt} = \frac{5}{4\pi} = \frac{35}{88}$ m/h $\left(\pi = \frac{22}{7}\right)$

Thus, the rate of change of water level is $\frac{35}{88}$ m/h.

26.

$$I = \int_0^{\frac{\pi}{2}} (2 \log \sin x - \log \sin 2x)\, dx$$

$$= \int_0^{\frac{\pi}{2}} (\log \sin^2 x - \log \sin 2x)\, dx$$

$$= \int_0^{\frac{\pi}{2}} \log\left(\frac{\sin^2 x}{\sin 2x}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \log\left(\frac{\sin^2 x}{2 \sin x\cdot\cos x}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \log\left(\frac{\tan x}{2}\right) dx$$

$$I = \int_0^{\frac{\pi}{2}} (\log(\tan x) - \log 2)\, dx$$

$$= \int_0^{\frac{\pi}{2}} \log(\tan x)\, dx - \log 2\int_0^{\frac{\pi}{2}} 1\, dx$$

$$I = I_1 - \log 2\, (x)_0^{\frac{\pi}{2}}$$

$$I = I_1 - \frac{\pi}{2}\log 2 \qquad \ldots (1)$$

Now, $I_1 = \int_0^{\frac{\pi}{2}} \log(\tan x)\, dx$

By property (6),

$x = \frac{\pi}{2} - x$

$$I_1 = \int_0^{\frac{\pi}{2}} \left(\log \tan\left(\frac{\pi}{2} - x\right)\right) dx$$

$$= \int_0^{\frac{\pi}{2}} (\log \cot x)\, dx$$

$$= -\int_0^{\frac{\pi}{2}} \log(\tan x)\, dx$$

$I_1 = -I_1$

$\therefore\ 2I_1 = 0$

$\therefore\ I_1 = 0$

Put the value of I_1 in equation (1),

$$I = \frac{-\pi}{2}\log 2 \text{ OR } \frac{\pi}{2}\log\frac{1}{2}$$

27.

$$y(e)^{\frac{x}{y}}\, dx = \left(x(e)^{\frac{x}{y}} + y^2\right) dy$$

$$\therefore\ \frac{dx}{dy} = \frac{x(e)^{\frac{x}{y}} + y^2}{y(e)^{\frac{x}{y}}}$$

$$\therefore\ \frac{dx}{dy} = \frac{x}{y} + \frac{y}{(e)^{\frac{x}{y}}} \qquad \ldots (1)$$

Now, Take $\frac{x}{y} = v$,

$\therefore\ x = vy$

→ Differentiate w.r.t. y,

$\therefore\ \frac{dx}{dy} = v + y\frac{dv}{dy}$

→ Put this value of equation (1),

$v + y\frac{dv}{dy} = v + \frac{y}{e^v}$

$\therefore\ y\frac{dv}{dy} = \frac{y}{e^v}$

$\therefore\ e^v\, dv = dy$

$\therefore\ \int e^v\, dv = \int 1\, dy$

$\therefore\ e^v = y + c$

$\therefore\ (e)^{\frac{x}{y}} = y + c$

Which is required particular solution of given differential equation.